

Cambridge International AS & A Level

MATHEMATICS (9709) P3

TOPIC WISE QUESTIONS + ANSWERS | COMPLETE SYLLABUS







Chapter 7

Vectors





248. 9709_s20_qp_31 Q: 9

With respect to the origin O	the vertices of a triangle ABC	have position vectors
Will respect to the origin of	the vertices of a triangle ADC	mave position vectors

			
$OA = 2\mathbf{i} + 5\mathbf{k},$	$OB = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$	and $OC = \mathbf{i} + \mathbf{j}$	+ k

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Show that triangle	e <i>ABC</i> is isosce	eles.		
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(c)

Find the exact length of the perpendicular from O to the line through B and C .	[4]
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 $249.\ 9709_s20_qp_32\ Q:\ 10$

With respect to the origin O, the points A and B have position vectors given by $\overrightarrow{OA} = 6\mathbf{i} + 2\mathbf{j}$ and $\overrightarrow{OB} = 2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. The midpoint of OA is M. The point N lying on AB, between A and B, is such that AN = 2NB.

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The line through M and N intersects the line through O and B at the point P.

))	Find the position vector of <i>P</i> .	[3
`	Calculate angle <i>OPM</i> , giving your answer in degrees.	C.
,	Calculate angle OI M, giving your answer in degrees.	
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(a)

 $250.\ 9709_s20_qp_33\ Q:\ 8$

Relative to the origin O.	the points A	and D have	nosition	vootore cive	n h
Relative to the origin O .	ine boints A. I	and D nave	DOSILION	vectors give	n ov

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$$
, $\overrightarrow{OB} = 2\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OD} = 3\mathbf{i} + 2\mathbf{k}$.

A fourth point C is such that ABCD is a parallelogram.

Find the position vector of C and verify that the parallelogram is not a rhombus.	[5]
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(a)

 $251.\ 9709_w20_qp_31\ Q:\ 11$

Two lines have equations $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$.	, where a is a
constant.	

Given that the two lines intersect, find the value of a and the position vector of the point of intersection. [5]
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two	en instead that the a possible values of <i>a</i>					-
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252. 9709_w20_qp_32 Q: 8

With respect to the origin O, the position vectors of the points A, B, C and D are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}.$$

(a)	Show that $AB = 2CD$.
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(b)	Find the angle between the directions of \overrightarrow{AB} and \overrightarrow{CD} . [3]





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 $253.\ 9709_m19_qp_32\ Q:\ 9$

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Find the acute angle between the planes.	[4]
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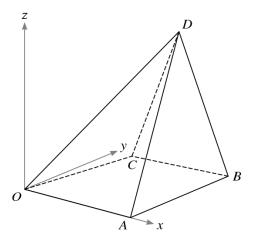


Find a vector equation for the line of intersection of the planes.	[6]
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 $254.\ 9709_s19_qp_31\ Q:\ 9$



The diagram shows a set of rectangular axes Ox, Oy and Oz, and four points A, B, C and D with position vectors $\overrightarrow{OA} = 3\mathbf{i}$, $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$, $\overrightarrow{OC} = \mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{OD} = 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$.

Find the equation of the plane BCD , giving your answer in the form $ax + by + cz = d$. [6]
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 $255.\ 9709_s19_qp_32\ Q:\ 9$

The points A and B have position vectors $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ respectively. The line l has equal $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.	tion
(i) Show that l does not intersect the line passing through A and B .	[5]

Show that l does not intersect the line passing through A and B .	[5]
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merseets tile	line l at the point l	P. Find the e	quation of n	and the pos	sition vector	1 OI P.
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 $256.\ 9709_s19_qp_33\ Q{:}\ 10$

The line *l* has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$.

The point P has position vector $4\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$. Find the length of the perpendicular from P to l.
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It is given that l lies in the plane with equation $ax + by + 2z = 13$, where a and b are constant Find the values of a and b .
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 $257.\ 9709_w19_qp_31\ Q:\ 7$

Two lines l and m have equations $\mathbf{r} = a\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$ respectively, where a is a constant. It is given that the lines intersect.

(i)	Find the value of <i>a</i> .	[4]
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When a has this value, find the equation of the plane containing l and m .	
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 $258.\ 9709_w19_qp_33\ Q:\ 7$

The plane <i>m</i> has equation $x + 4y - 8z = 2$.	The plane n is parallel to m and passes through the po	int
P with coordinates $(5, 2, -2)$.		

(i)	Find the equation of n , giving your answer in the form $ax + by + cz = d$.	[2]
)
(ii)	Calculate the perpendicular distance between m and n .	[3]
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(iii)	The line l lies in the plane n , passes through the point P and is perpendicular to OP , where O is the origin. Find a vector equation for l . [4]		





 $259.\ 9709_m18_qp_32\ Q:\ 10$

		2x - 3y - z = 4.
(i)) Find the position vector of the point of intersection of l and p .	[3]
		.0,
(ii)) Find the acute angle between l and p .	[3]





4 j – k . F	d plane q is parallel to l , perpendicular to p and contains the point within the equation of q , giving your answer in the form $ax + by + cz = a$	<i>1</i> .
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 $260.\ 9709_s18_qp_31\ \ Q:\ 10$

he	point <i>P</i> has position vector $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. The line <i>l</i> has equation $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.
(i)	Find the length of the perpendicular from P to l , giving your answer correct to 3 significan figures. [5]
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	Find the equation of the plane containing l and P , giving your answer in the form $ax + by + cz$
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 $261.\ 9709_s18_qp_32\ \ Q:\ 10$

Two lines l and m have equations $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \mathbf{k} + s(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + t(\mathbf{i} + 2\mathbf{j} + 3\mathbf{j} + 4\mathbf{k})$	- k)
respectively.	

(i)) Show that the lines are skew.	[4]
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A pl	plane p is parallel to the lines l and m .	
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(11)	Find a vector that is normal to p.	[3]
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(iii)	Given that p is equidistant from the lines l and m , find the equation of p . Give your answer in the form $ax + by + cz = d$. [3]
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262. $9709_s18_qp_33$ Q: 10

The points A and B have position vectors $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $4\mathbf{i} + \mathbf{j} + \mathbf{k}$	respectively. The line l has equation
$\mathbf{r} = 4\mathbf{i} + 6\mathbf{j} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}).$	

Show that l does not intersect the line passing through A and B .	[5]
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The point P, with parameter t, lies on l and is such that angle PAB is equal to 120° .

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 $263.\ 9709_w18_qp_31\ Q:\ 10$ 

The planes m and n have equations $3x + y - 2z = 10$ and $x - 2y + 2z = 5$ respectively.	The line $l$ has
equation $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ .	

<b>(i)</b>	Show that $l$ is parallel to $m$ .	[3]
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(ii)	Calculate the acute angle between the planes $m$ and $n$ .	[3]
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the position	is on the line $l$ . The perpendicular distance of $P$ from the plane $n$ is equal to 2. vectors of the two possible positions of $P$ .
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 $264.\ 9709_w18_qp_32\ Q:\ 10$ 

The line l has equation  $\mathbf{r} = 5\mathbf{i} - 3\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ . The plane p has equation  $(\mathbf{r} - \mathbf{i} - 2\mathbf{j}) \cdot (3\mathbf{i} + \mathbf{j} + \mathbf{k}) = 0$ .

The line l intersects the plane p at the point A.

(i)	Find the position vector of $A$ .	[3]
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Calculate the acute angle between $l$ and $p$ .	
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 $265.\ 9709_m17_qp_32\ Q:\ 6$ 

Show that the line $l$ lies in the plane $p$ .	
show that the fine $i$ lies in the plane $p$ .	
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$3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ .	lane is parallel t Find the equation	n of this plan	e, giving yo	ur answer i	n the form <i>ax</i>	x + by + cz = d
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266. 9709_s17_qp_31 Q: 6

(i)

The plane with equation 2x + 2y - z = 5 is denoted by m. Relative to the origin O, the points A and B have coordinates (3, 4, 0) and (-1, 0, 2) respectively.

Show that the plane $m$ bisects $AB$ at right angles.	[5]
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A second plane p is parallel to m and nearer to O. The perpendicular distance between the planes is 1. (ii) Find the equation of p, giving your answer in the form ax + by + cz = d. [3]





267. 9709_s17_qp_32 Q: 9

Relative to the origin $O$ , the point $A$ has position	vector given by	$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 4\mathbf{k}.$	The line $l$ has
equation $\mathbf{r} = 9\mathbf{i} - \mathbf{j} + 8\mathbf{k} + \mu(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}).$			

) Find the po vector of the	sition vector e reflection o	of the foot of $A$ in $l$ .	of the perp	pendicular	from $A$ to $b$	. Hence 1	and the po	ositior [5]
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			9.		
Find the exac	t value of the perper	ndicular distance	of A from this pl	lane.	
	20.				
	AV.	r.			
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 $268.\ 9709_s17_qp_33\ \ Q:\ 10$ 

The points A and B have position vectors given by $\overrightarrow{OA}$	$\overrightarrow{l} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$ . The line $l$
has equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + m\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} - 4\mathbf{k})$ , where m	is a constant.

	he line $l$ intersects the line passing through $A$ and $B$ , find the value of $m$ .	
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Give you	answer in the form $ax + by + cz = d$ .	
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 $269.\ 9709_w17_qp_31\ \ Q:\ 10$ 

The equations of two lines $l$ and $m$ are $\mathbf{r} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	2k)
respectively.	

(i)	Show that the lines do not intersect.	[3]
	*.(	
<b>(**</b> )		Fo
(11)	Calculate the acute angle between the directions of the lines.	[3
	**	





Find the equation of the plane which passe to both $l$ and $m$ . Give your answer in the fo	$31111 \ ax + by + cz = a.$	[3]
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A ^c		•••••
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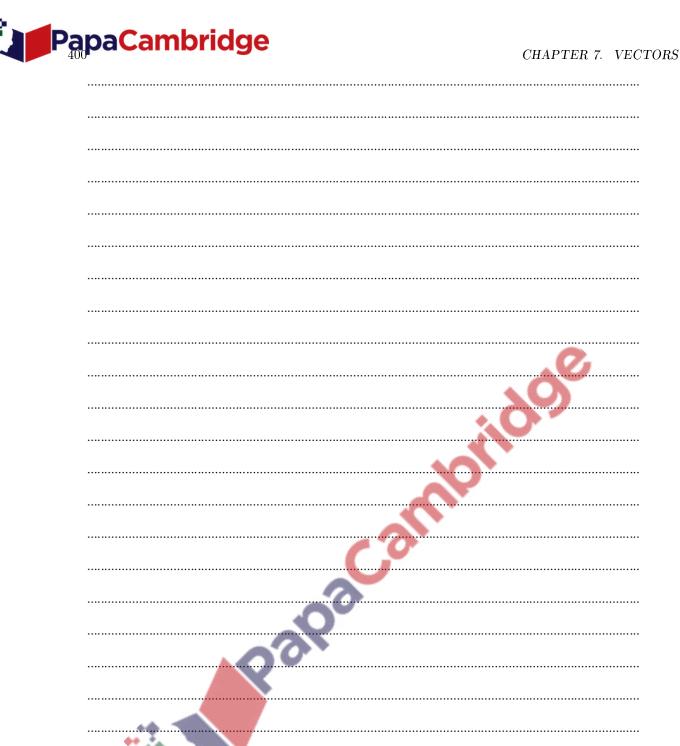


 $270.\ 9709_w17_qp_32\ Q{:}\ 10$ 

Calculate the acute angle	be between the planes $p$ and $q$ .	
		.0
		10)
		.0
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	40	
The point $A$ on the line of the plane which contains	of intersection of $p$ and $q$ has $y$ -coordinate $\epsilon$ ins the point $A$ and is perpendicular to be	equal to 2. Find the equal to $p$ and $p$ and $p$ .
your answer in the form	ax + by + cz = d.	
***	7	
•		









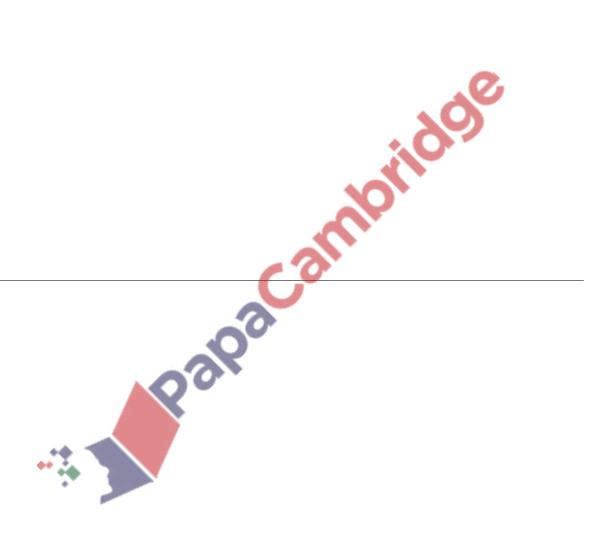
[3]



 $271.\ 9709_m16_qp_32\ Q:\ 8$ 

The line *l* has equation  $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ . The plane *p* has equation  $\mathbf{r} \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = 6$ .

- (i) Show that l is parallel to p.
- (ii) A line m lies in the plane p and is perpendicular to l. The line m passes through the point with coordinates (5, 3, 1). Find a vector equation for m.





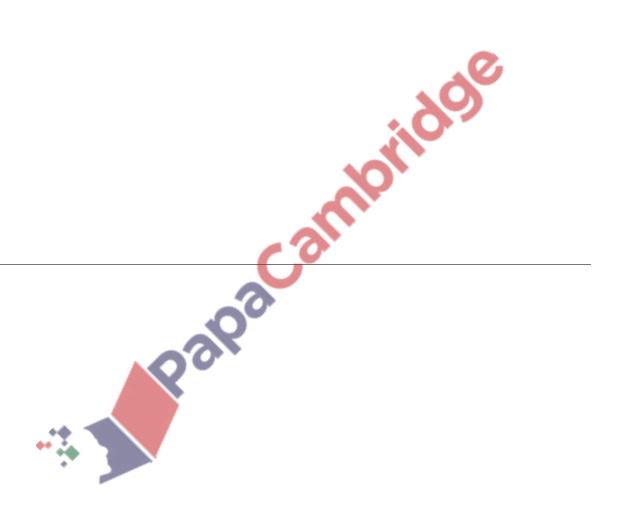


 $272.\ 9709_s16_qp_31\ \ Q:\ 9$ 

With respect to the origin O, the points A, B, C, D have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$$
,  $\overrightarrow{OB} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ ,  $\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ ,  $\overrightarrow{OD} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ .

- (i) Find the equation of the plane containing A, B and C, giving your answer in the form ax + by + cz = d. [6]
- (ii) The line through D parallel to OA meets the plane with equation x + 2y z = 7 at the point P. Find the position vector of P and show that the length of DP is  $2\sqrt{14}$ .



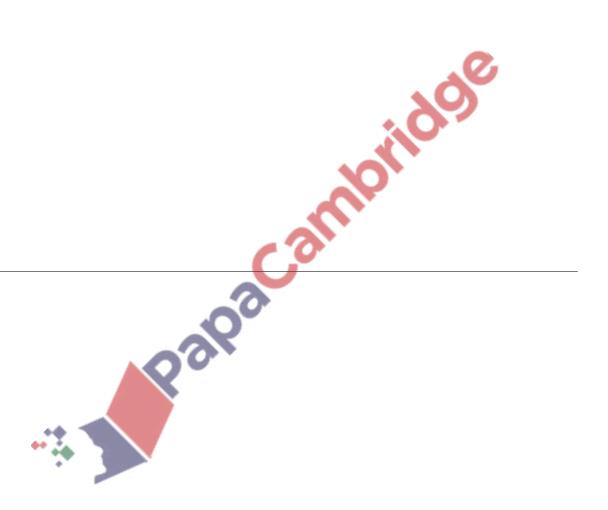




 $273.\ 9709 \ \ s16 \ \ qp \ \ 32 \ \ Q:\ 9$ 

The points A, B and C have position vectors, relative to the origin O, given by  $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ ,  $\overrightarrow{OB} = 4\mathbf{j} + \mathbf{k}$  and  $\overrightarrow{OC} = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$ . A fourth point D is such that the quadrilateral ABCD is a parallelogram.

- (i) Find the position vector of D and verify that the parallelogram is a rhombus. [5]
- (ii) The plane p is parallel to OA and the line BC lies in p. Find the equation of p, giving your answer in the form ax + by + cz = d. [5]





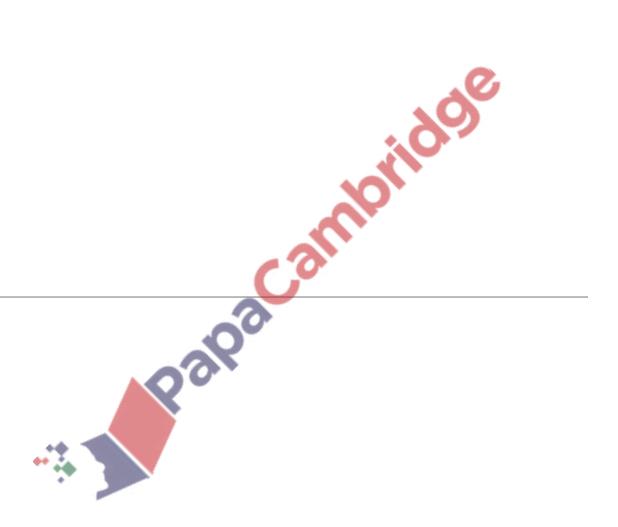


 $274.\ 9709_s16_qp_33\ Q:\ 8$ 

The points A and B have position vectors, relative to the origin O, given by  $\overrightarrow{OA} = \mathbf{i} + \mathbf{j} + \mathbf{k}$  and  $\overrightarrow{OB} = 2\mathbf{i} + 3\mathbf{k}$ . The line l has vector equation  $\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} - \mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ .

(i) Show that the line passing through A and B does not intersect I. [4]

(ii) Show that the length of the perpendicular from A to l is  $\frac{1}{\sqrt{2}}$ . [5]



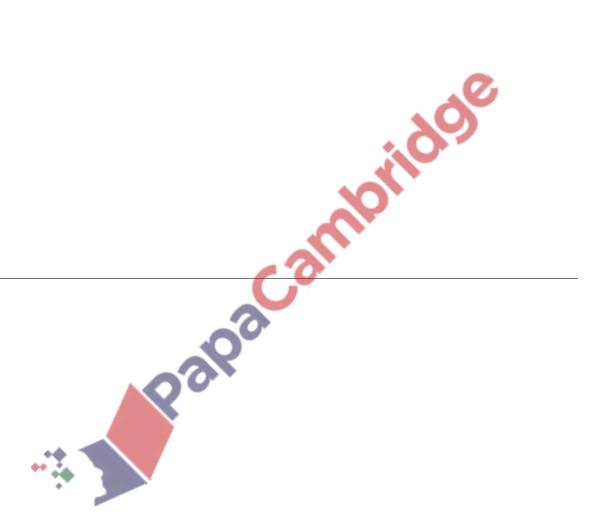




 $275.\ 9709_w16_qp_31\ Q:\ 8$ 

Two planes have equations 3x + y - z = 2 and x - y + 2z = 3.

- (i) Show that the planes are perpendicular. [3]
- (ii) Find a vector equation for the line of intersection of the two planes. [6]



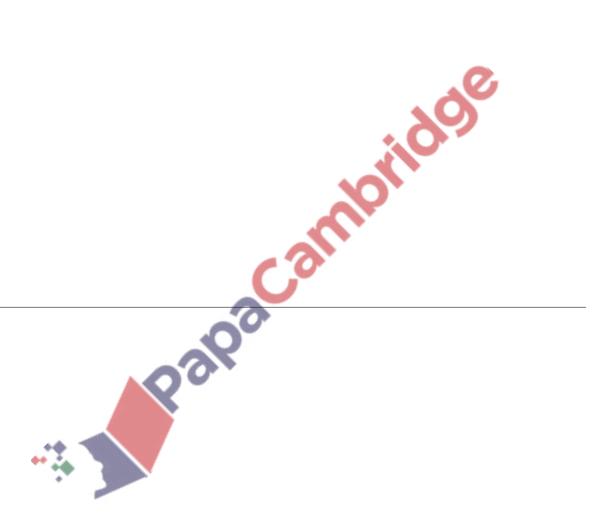




 $276.\ 9709_w16_qp_33\ Q:\ 10$ 

The line *l* has vector equation  $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + \mathbf{k})$ .

- (i) Find the position vectors of the two points on the line whose distance from the origin is  $\sqrt{(10)}$ .
- (ii) The plane p has equation ax + y + z = 5, where a is a constant. The acute angle between the line l and the plane p is equal to  $\sin^{-1}(\frac{2}{3})$ . Find the possible values of a. [5]



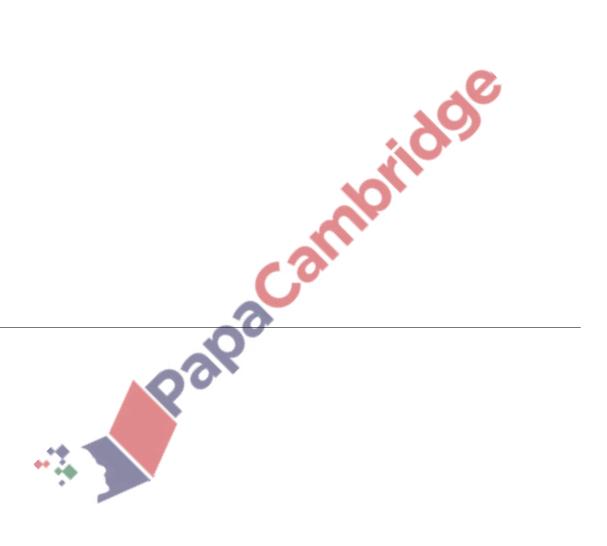




277. 9709_s15_qp_31 Q: 6

The straight line  $l_1$  passes through the points (0, 1, 5) and (2, -2, 1). The straight line  $l_2$  has equation  $\mathbf{r} = 7\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$ .

- (i) Show that the lines  $l_1$  and  $l_2$  are skew. [6]
- (ii) Find the acute angle between the direction of the line  $l_2$  and the direction of the x-axis. [3]





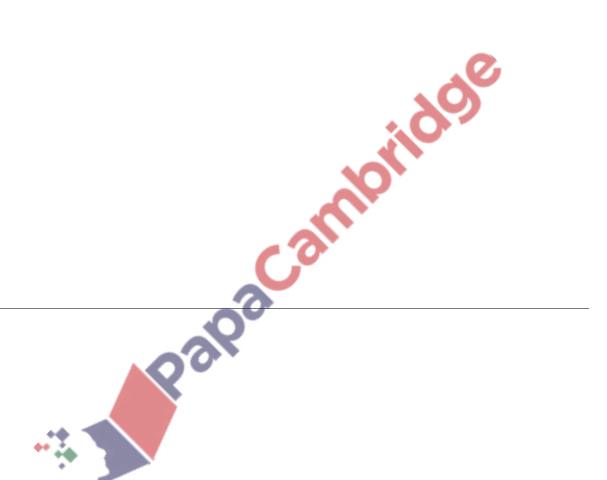
[5]



 $278.\ 9709_s15_qp_32\ Q:\ 10$ 

The points A and B have position vectors given by  $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$  and  $\overrightarrow{OB} = \mathbf{i} + \mathbf{j} + 5\mathbf{k}$ . The line l has equation  $\mathbf{r} = \mathbf{i} + \mathbf{j} + 2\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} - \mathbf{k})$ .

- (i) Show that l does not intersect the line passing through A and B.
- (ii) Find the equation of the plane containing the line l and the point A. Give your answer in the form ax + by + cz = d. [6]







 $279.\ 9709_s15_qp_33\ Q:\ 9$ 

Two planes have equations x + 3y - 2z = 4 and 2x + y + 3z = 5. The planes intersect in the straight line I

(i) Calculate the acute angle between the two planes. [4]

(ii) Find a vector equation for the line l. [6]







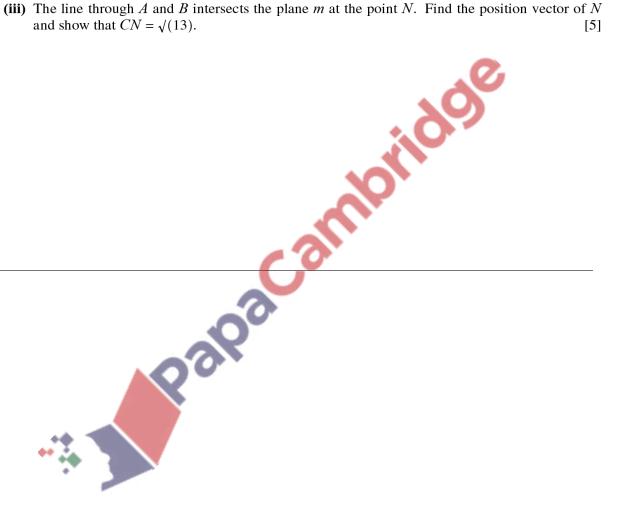
280.  $9709_{y15_{qp}31} Q: 7$ 

The points A, B and C have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}.$$

The plane m is perpendicular to AB and contains the point C.

- (i) Find a vector equation for the line passing through A and B. [2]
- (ii) Obtain the equation of the plane m, giving your answer in the form ax + by + cz = d. [2]
- (iii) The line through A and B intersects the plane m at the point N. Find the position vector of Nand show that  $CN = \sqrt{13}$ . [5]







A plane has equation 4x - y + 5z = 39. A straight line is parallel to the vector  $\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$  and passes through the point A(0, 2, -8). The line meets the plane at the point B.

- (i) Find the coordinates of B. [3]
- (ii) Find the acute angle between the line and the plane. [4]
- (iii) The point C lies on the line and is such that the distance between C and B is twice the distance between A and B. Find the coordinates of each of the possible positions of the point C. [3]

